BENDING OF AXISYMMETRIC PLATES

In an axisymmetric analysis, plates may be used as circular plates. This document describes an example that has been used to verify the bending of the axisymmetric plates in PLAXIS 2D. This verification example involve a uniformly distributed load \( q \) on a circular plate. In one case the plate can rotate freely at the boundary and in the other case the plate is clamped, as indicated in Figure 1.

![Figure 1 Loading scheme for testing axisymmetric plates](image)

Figure 1 Loading scheme for testing axisymmetric plates

Used version:
- PLAXIS 2D - Version 2011

**Input:** Beams cannot be used individually. A cluster is used to create the geometry (Figure 2).

![Figure 2 Geometries of the projects](image)

Figure 2 Geometries of the projects

**Materials:** The properties, the dimensions and the load of the beam are:

\[
\begin{align*}
EA &= 1200 \text{ kN} \\
EI &= 1 \text{ kN m}^2 \\
\nu &= 0.0 \\
l &= 1 \text{ m} \\
q &= 1 \text{ kN/m}
\end{align*}
\]

**Meshing:** The *Medium* option is selected for the *Element distribution* of the *Global coarseness*.

**Calculations:** In the Initial phase zero initial stresses are generated by using the K0 procedure with \( \Sigma - Mweight \) equal to zero. A new calculation phase is introduced (Phase 1) and the *Calculation type* is set to *Plastic analysis*. The *Reset displacements to zero* is selected and the *Tolerated error for Iterative procedure* is set to 0.001. In this phase the soil clusters are deactivated and the plates are activated.

**Output:** The results of the two calculations are plotted in Figure 3 and Figure 4.

**Verification:** For the situation of a circular plate with a uniformly distributed load one can elaborate and solve a differential equation. The analytical solutions for this equation
depend on the boundary conditions.

For the plate with free rotation at the boundary one finds:

Settlement:

\[
w = \frac{q R^4}{64 D} \left( \frac{5 + \nu}{1 + \nu} - \frac{6 + 2\nu}{1 + \nu} \frac{r^2}{R^2} + \frac{r^4}{R^4} \right)
\]

Moments:

\[
m_{rr} = \frac{q R^2}{16} \left( (3 + \nu) - (3 + \nu) \frac{r^2}{R^2} \right)
\]

\[
m_{tt} = \frac{q R^2}{16} \left( (3 + \nu) - (1 + 3\nu) \frac{r^2}{R^2} \right)
\]

<table>
<thead>
<tr>
<th>Location</th>
<th>Analytical solution</th>
<th>Numerical solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r = 0)</td>
<td>(w = 0.078125) m (m_{rr} = 0.18750) kNm/m</td>
<td>(w = 0.078625) m (m_{rr} = 0.187484) kNm/m</td>
</tr>
<tr>
<td>(r = R/2)</td>
<td>(w = 0.055664) m (m_{rr} = 0.140625) kNm/m</td>
<td>(w = 0.056039) m (m_{rr} = 0.140625) kNm/m</td>
</tr>
</tbody>
</table>

For the plate with a clamped boundary one finds:

Settlement:

\[
w = \frac{q R^4}{64 D} \left( 1 - \frac{r^2}{R^2} \right)^2
\]

Moments:

\[
m_{rr} = \frac{q R^2}{16} \left( (1 + \nu) - (3 + \nu) \frac{r^2}{R^2} \right)
\]

\[
m_{tt} = \frac{q R^2}{16} \left( (1 + \nu) - (1 + 3\nu) \frac{r^2}{R^2} \right)
\]
### Location | Analytical solution | Numerical solution
--- | --- | ---
$r = 0$ | $w = 0.01563 \text{ m}$  
$m_{rr} = 0.06250 \text{ kNm/m}$ | $w = 0.016125 \text{ m}$  
$m_{rr} = 0.062503 \text{ kNm/m}$
$r = R/2$ | $w = 0.008789 \text{ m}$  
$m_{rr} = 0.015625 \text{ kNm/m}$ | $w = 0.0.009164 \text{ m}$  
$m_{rr} = 0.015625 \text{ kNm/m}$
$r = R$ | $m_{rr} = -0.125 \text{ kNm/m}$ | $m_{rr} = -0.125 \text{ kNm/m}$

The settlement difference is mainly due to shear deformation, which is included in the numerical solution but not in the analytical solution. Apart from this, the numerical results are very close to the analytical solution.