PERFORMANCE OF SHELL ELEMENTS

Tunnel lining can be modelled as a beam in PLAXIS 2D and as a plate in PLAXIS 3D. By using this element, 3 types of deformations are taken into account: shear deformation, compression due to normal forces and obviously bending. This document describes an example that has been used to verify the shell elements in PLAXIS.

Used version:

- PLAXIS 2D - Version 2011
- PLAXIS 3D - Version 2012

Input: For the 2D model a ring with a radius of R = 1 m is considered. In order to model such a ring the bottom point of the ring is fixed with respect to translation and rotation and the top point is allowed to move only in the vertical direction. The load F = 1 kN/m is applied only at the top point. Geometric non-linearity is not taken into account.

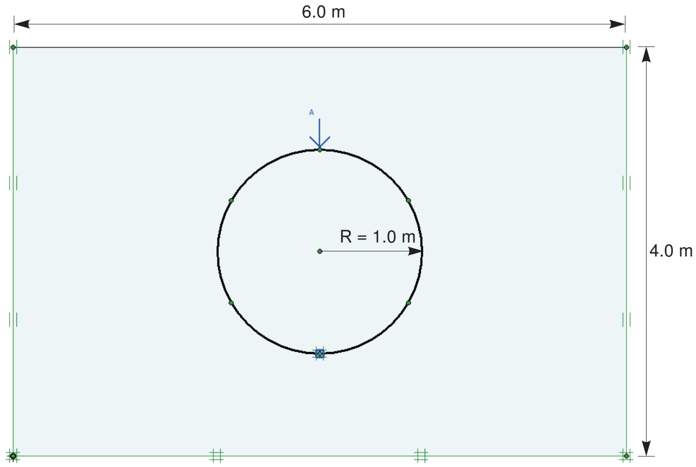


Figure 1 Project layout (PLAXIS 2D)

For the 3D model a 4x4x0.15 m soil volume is considered. A cylinder with a radius of R = 1 m is imported. The layout of the geometry is displayed in Figure 2. A line prescribed displacement is defined at the bottom of the cylinder, fixing it with respect to translation and rotation and the top point is allowed to move only in the vertical direction. The load F = 1 kN/m is applied only at the top point. Geometric non-linearity is not taken into account. Two surfaces are created dividing the model in three in the y direction (Figure 2).

Materials: The Young's modulus and the Poisson's ratio of the cluster material are taken respectively as $E = 10^6$ kPa and $\nu = 0$. The Young's modulus of the shell material is taken as $E = 10^6$ kPa. For the thickness of the ring cross section, H, several different values are considered so that we have rings ranging from very thin to very thick (H = 0.01, 0.02, 0.05, 0.1, 0.2, 0.5).

Meshing: The *Medium* and *Fine* options are selected for the *Global coarseness* for PLAXIS 2D and PLAXIS 3D respectively.

Calculations: In the Initial phase zero initial stresses are generated by using the K0

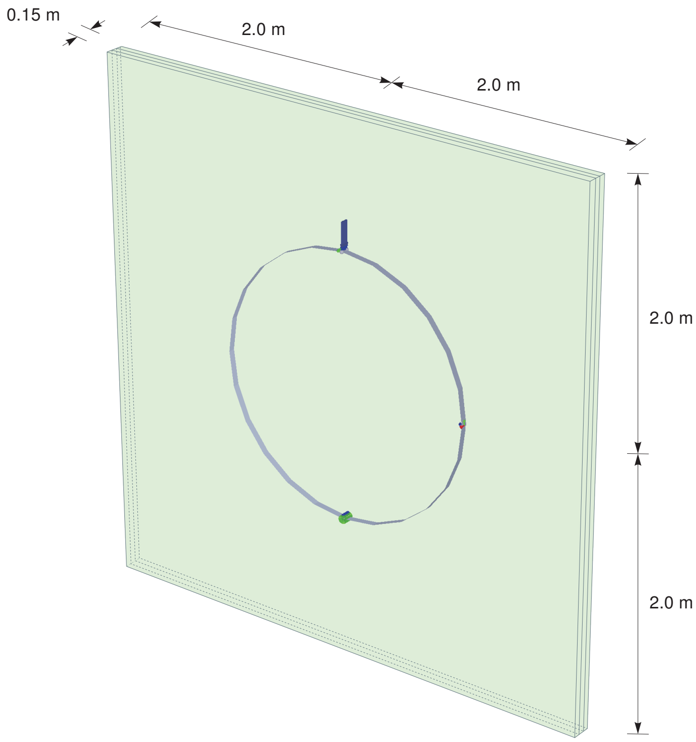


Figure 2 Project layout (PLAXIS 3D)

Table 1 Material properties for plates (PLAXIS 2D)

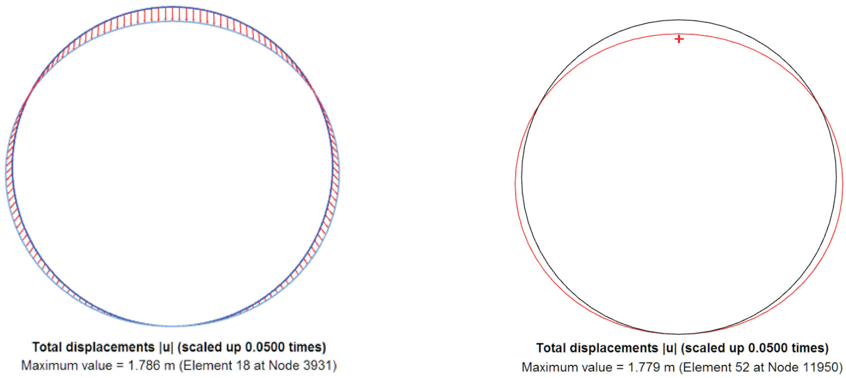
H	EA	I	EI
0.01	$1{\cdot}10^{4}$	$1{\cdot}10^{-7}$	0.08
0.02	$2{\cdot}10^{4}$	$7{\cdot}10^{-7}$	0.67
0.05	$5{\cdot}10^{4}$	$1.04{\cdot}10^{-5}$	10.42
0.1	$1{\cdot}10^{5}$	$8.33{\cdot}10^{-5}$	83.33
0.2	$2{\cdot}10^{5}$	$6.667{\cdot}10^{-4}$	667.67
0.5	$5{\cdot}10^{5}$	$1.04167{\cdot}10^{-2}$	10416.67

procedure with Σ -*Mweight* equal to zero. The shell elements are activated in a separate phase (phase 1 - 6). A separate material data set is defined for each of the phases (Table 1). The calculation type is *Plastic analysis* and a *Tolerated error* of 0.001 is defined.

Output: The deformed shape of the ring is shown in Figure 3.

The calculated normal force at the belly of the ring is 0.50 for all different values of ring thickness. The calculated bending moment at the belly varies from 0.182 to 0.188 as the ring changes from thin to thick. Typical graphs of the bending moment and normal force are shown in Figure 4.

Verification: The analytical solution for the deflection of the ring is given by Blake (1959), and the analytical solution for the bending moment and the normal force can be found from Roark (1965). The vertical displacement at the top of the ring is given by the following formula:



a. PLAXIS 2D   b. PLAXIS 3D

Figure 3 Deformed shape of the ring (H = 0.01)

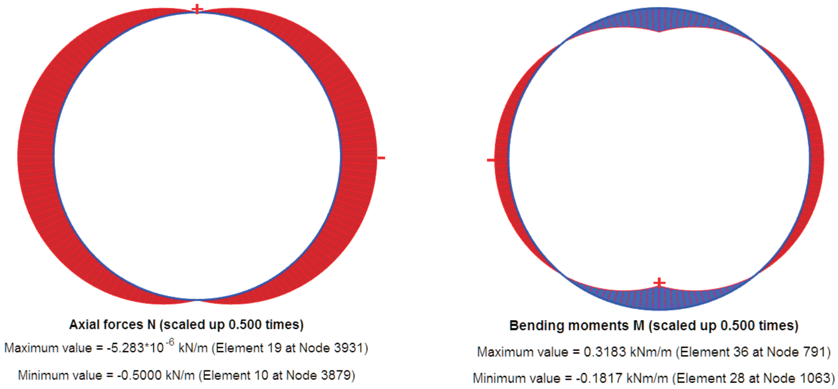


Figure 4 Resulting normal forces and bending moments (H = 0.01)(PLAXIS 2D)

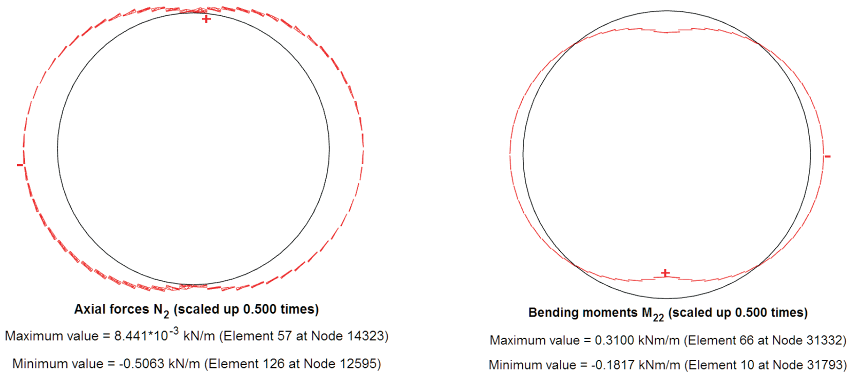


Figure 5 Resulting normal forces and bending moments (H = 0.01) (PLAXIS 3D)

$$\frac{F\lambda}{E}\left[1.788\,\lambda^2 + 3.091 - \frac{0.637}{1 + 12\,\lambda^2}\right] \qquad \text{where} \qquad \lambda = \frac{R}{H}$$

The resulting vertical deflections at the top point and the theoretical values are presented in Figure 6.

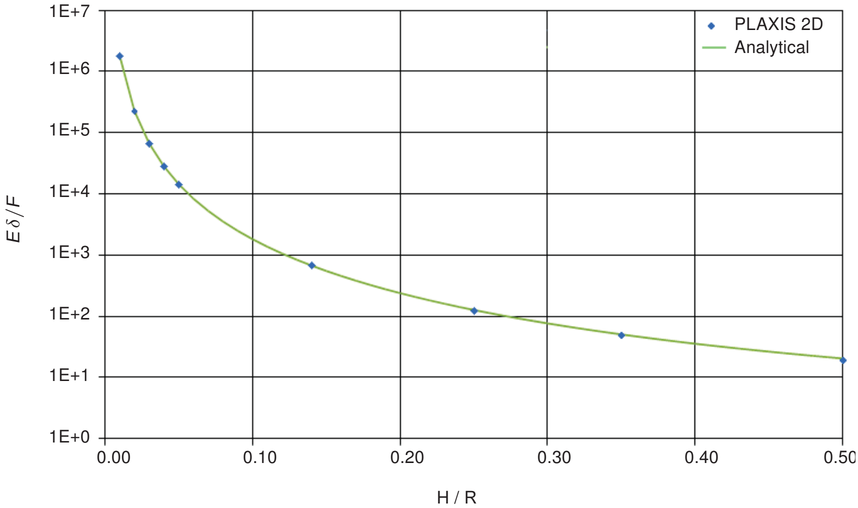


Figure 6 Calculated deflections compared with analytical solutions (PLAXIS 2D)

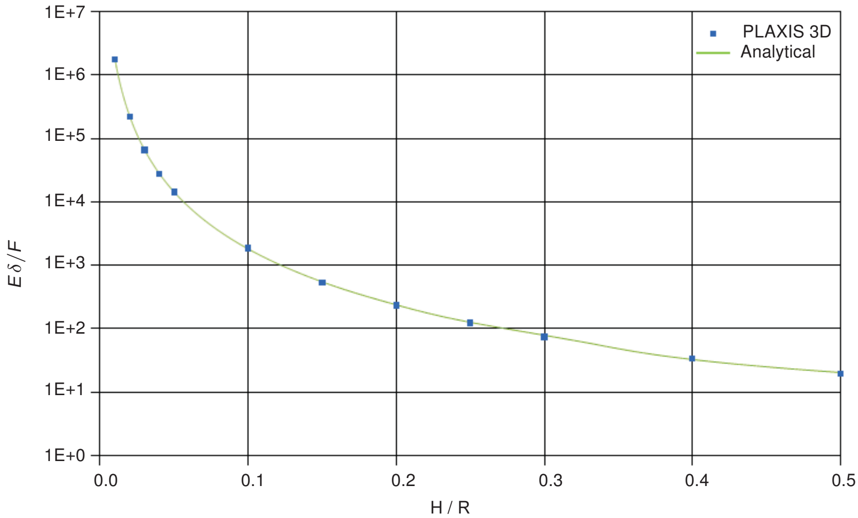


Figure 7 Calculated deflections compared with analytical solutions (PLAXIS 3D)

It can be seen that the deflections calculated by PLAXIS fit the theoretical solutions very well. Only for a very thick ring ($H/R = 0.5$) some errors are observed, which is about 6.06% for PLAXIS 2Dand about 4.44% for PLAXIS 3D. For thin rings the error is nearly zero.

The analytical solution for the bending moment and normal force at the belly is 0.182 and 0.5 respectively. Thus even for very thick rings the error in the bending moment is just 4% and 0.05%, and the error in the normal force is only 0.04% and 0.08% for PLAXIS 2D and PLAXIS 3D respectively.

REFERENCES

[1] Blake, A. (1959). Deflection of a thick ring in diametral compression. Am. Soc. Mech. Eng., J. Appl. Mech., 26(2).

[2] Roark, R.J. (1965). Formulas for Stress and Strain. McGraw-Hill Book Company.